Signals, systems, acoustics and the ear

Week 2

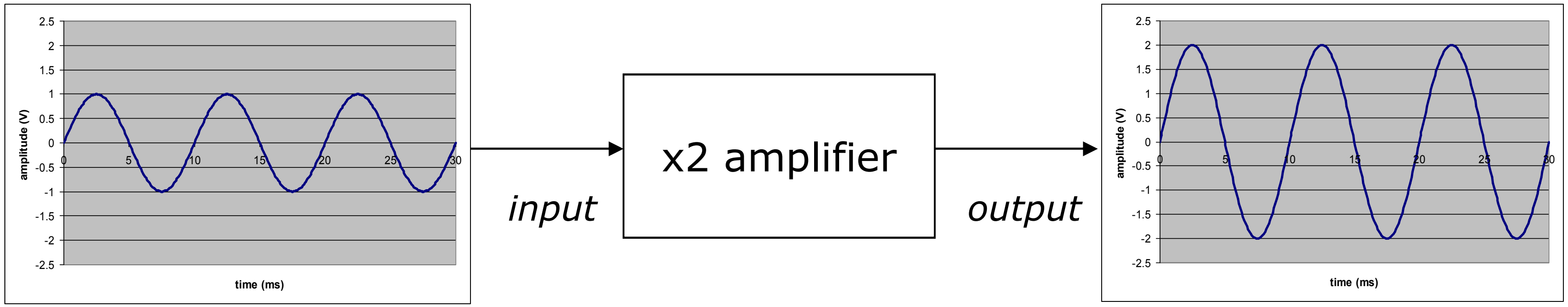
(Signals &) **Systems**
&
The ***Big*** idea

No eating or drinking in the lab!

Not even water

Reminder: What is a system?

- Something which performs an operation on, or transformation of, a signal
- For now, one input and one output



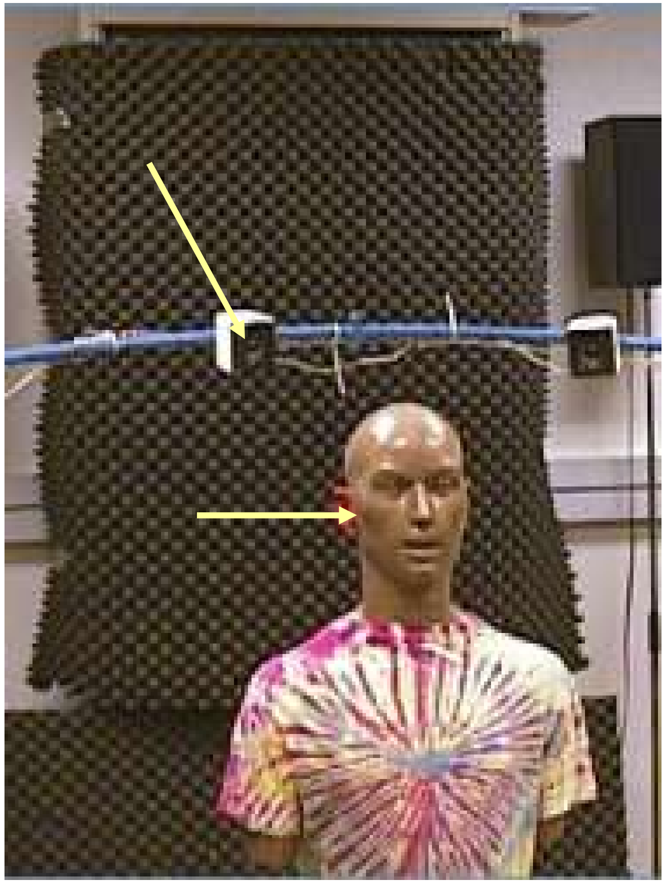
A microphone
(a special name for this kind of system?)

input = sound wave
(variations in pressure)

output = electrical wave
(variations in voltage)

System = body + head + pinna + ear canal

input = sound from a particular place in space



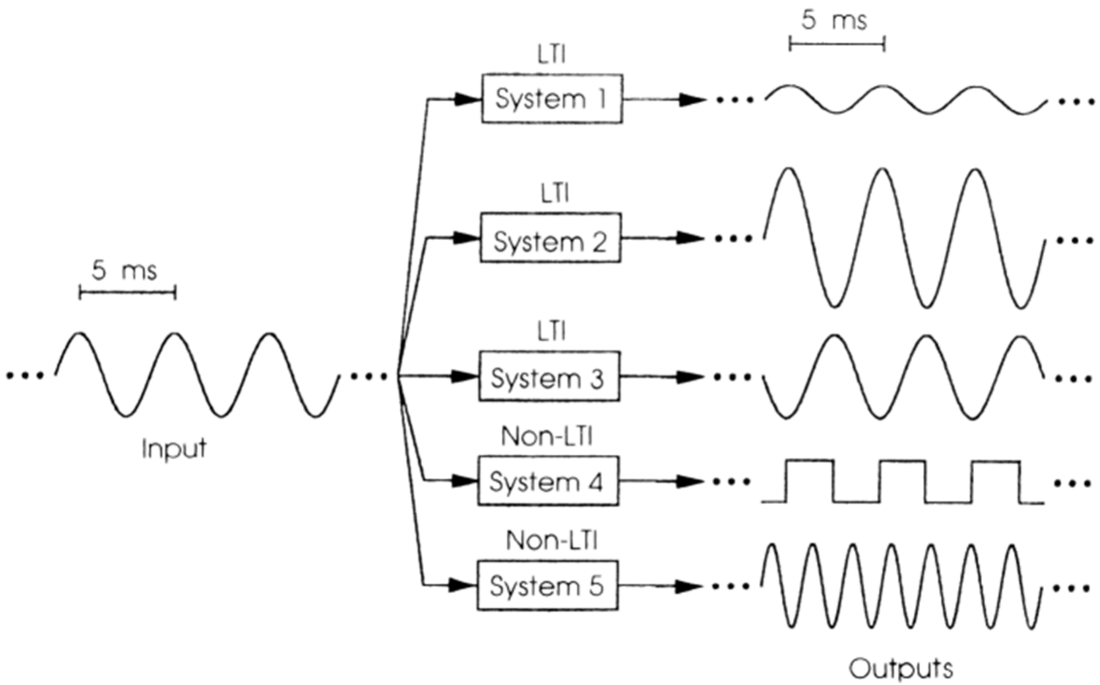
output = sound at the eardrum

The problem

- We want to be able to predict what a system will do to a wide variety of signals, without having to try each one.
  - For example, speech from different people through a hearing aid
- No solution for *all* possible systems.
- It *is* possible for a group of very special systems, known as *linear time-invariant (LTI) systems.*

Fact number 1

Sinusoidal input signals to an LTI system always lead to sinusoidal outputs of the **same frequency**



Fact number 2

An LTI system can be completely characterised by its response to sinusoids

Today’s Lab:
Measuring the frequency response of an acoustic resonator

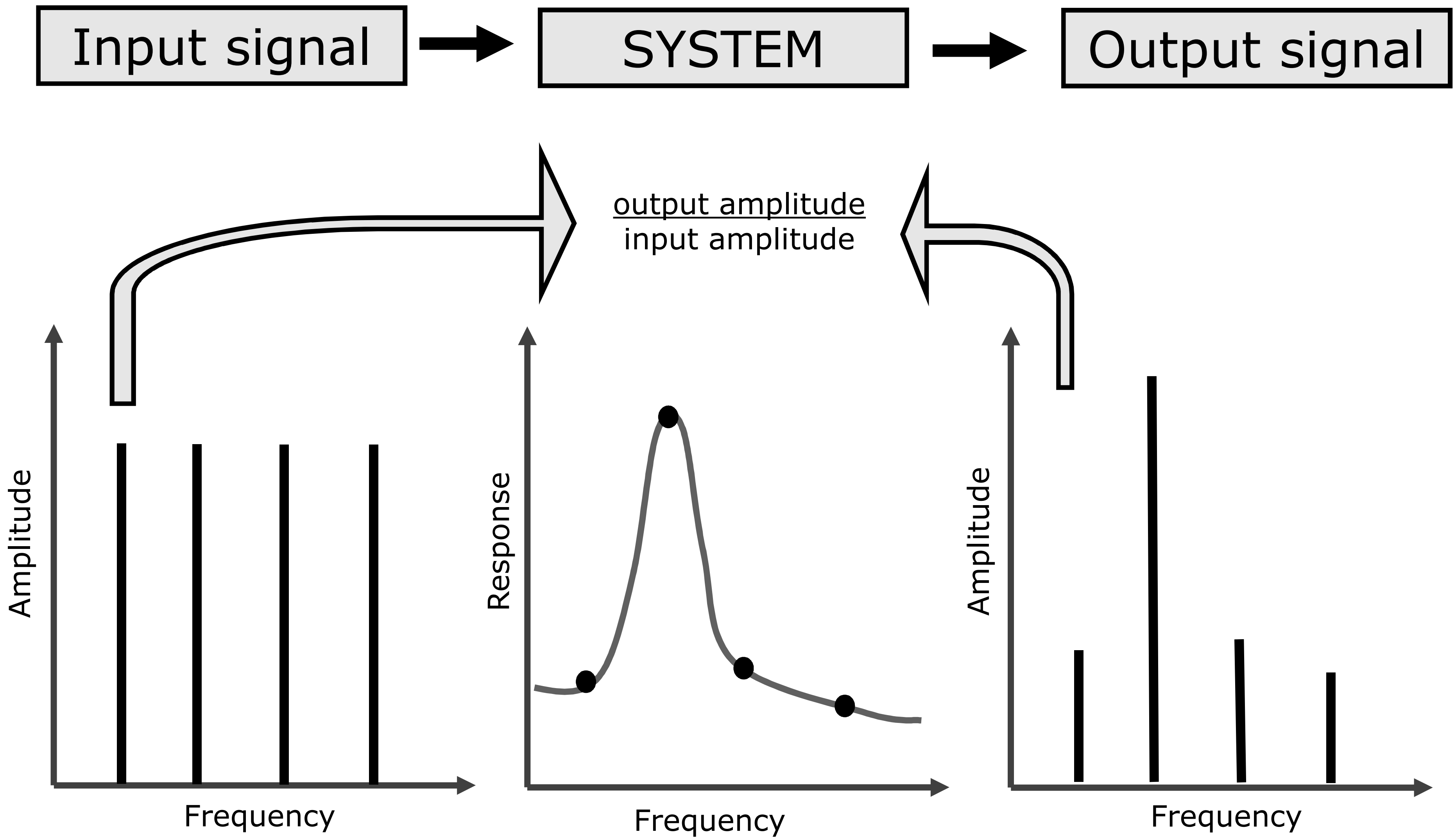
Frequency response

- Also known as a *transfer function*
- Sinusoids vary on 3 parameters
  - frequency, amplitude & phase
- For a system, we need to specify its effect on two of those
  - amplitude response
  - phase response
- Amplitude response typically more important …
  - but phase matters in certain situations
- So we will measure a so-called amplitude response.
  - How a system changes the amplitude of sinusoids
  - frequency response/transfer function/amplitude response

Using sinusoids to measure an amplitude response in an LTI system

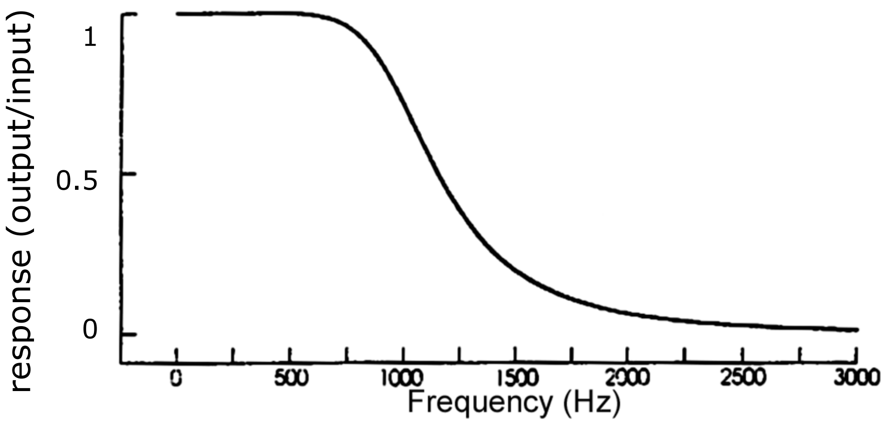
- Typically, choose a constant level for input (not necessary)
- For each frequency – feed the input sinusoid to the system and measure level at output
- Calculate the *response = output/input*
  - Also known as *gain*
- Need enough frequencies to map out amplitude response over frequency range of interest

Characterisation of LTI-Systems



At least 2 ways to specify a frequency response

frequency (Hz)	input (V)	output (V)	amplitude ratio (re 2V input)
250	2	2	1
500	2	1.98	0.99
1000	2	1.42	0.71
1500	2	0.56	0.28
2000	2	0.24	0.12
3000	2	0.08	0.04



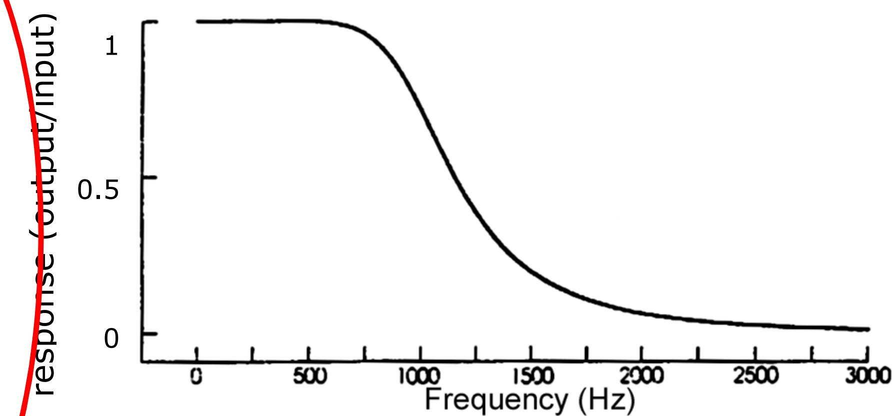
But easiest to see the overall effect on a graph, e.g. a low-pass response

Scaling the response

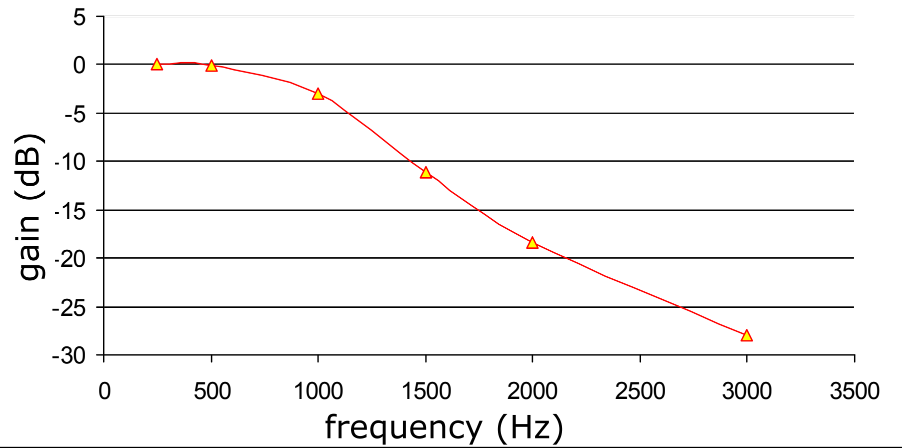
- Generally use a logarithmic scale for response (dB) rather than linear
- Amplitude ratio expressed in dB
  =20 x log(output amp/input amp)
- Note similarity to dB SPL
  - 20 log (? Pa/20 x 10^{-6} Pa)
- Expresses output level in dB re input level

At least 3 ways to specify a frequency response

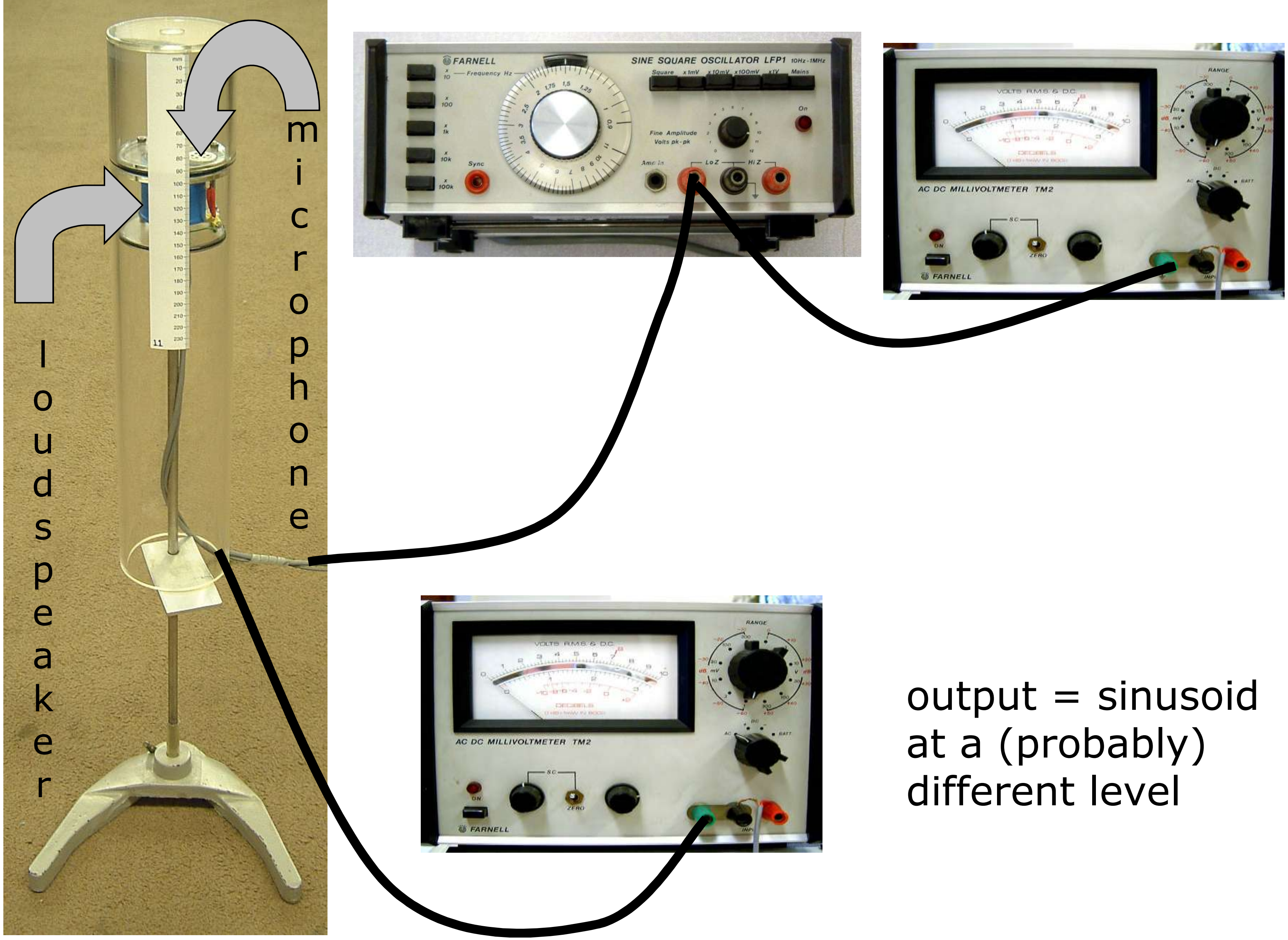
frequency (Hz)	input (V)	output (V)	amplitude ratio (re 2V input)	gain in dB
250	2	2	1	0.0
500	2	1.98	0.99	-0.1
1000	2	1.42	0.71	-3.0
1500	2	0.56	0.28	-11.1
2000	2	0.24	0.12	-18.4
3000	2	0.08	0.04	-28.0



But easiest to see the overall effect on a graph, e.g. a low-pass response



input = sinusoid from oscillator



output = sinusoid at a (probably) different level

Battle stations everyone!

Linear Time-Invariant (LTI) Systems

Linearity = Homogeneity + Additivity

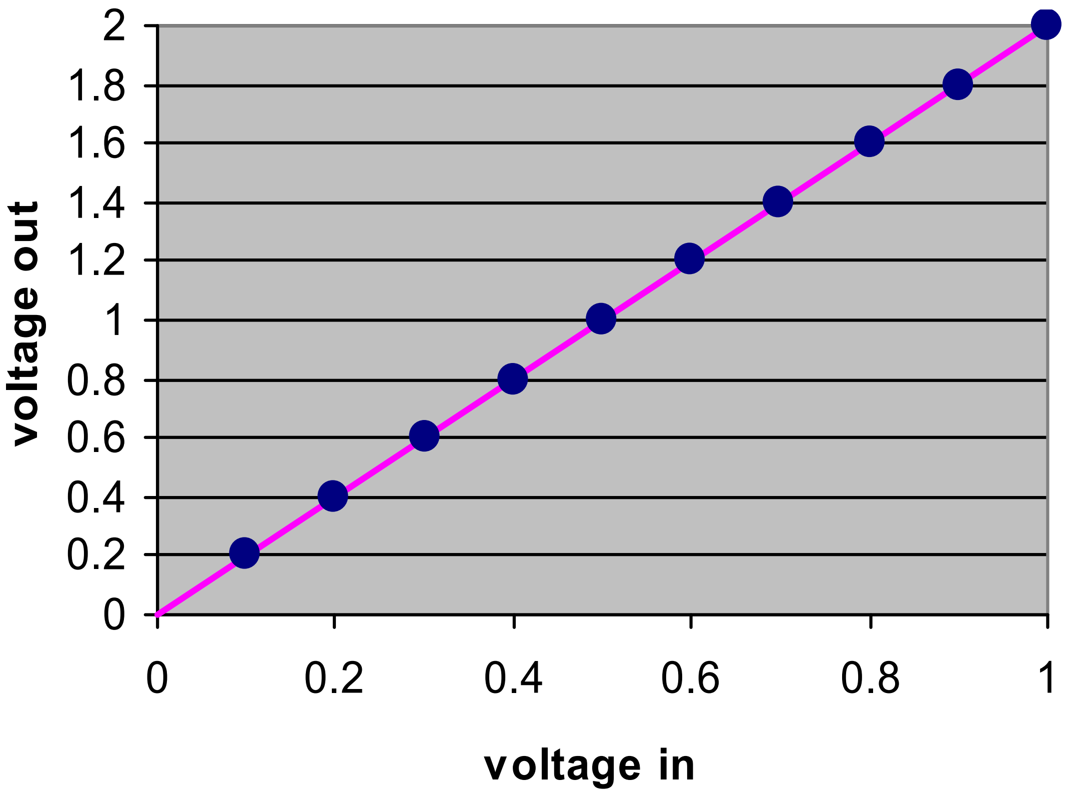
Linearity in a system: Homogeneity

- Homogeneity
  - for a particular pair of input and output signals, any change in the size of the input signal is matched by the same change in the size of the output
  - If $inp(t) \rightarrow outp(t)$
  - Then $k \cdot inp(t) \rightarrow k \cdot outp(t)$
- In other words …
  - Doubling the size of the input signal doubles the size of the output signal
  - Halving the size of the input signal halves the size of the output signal
- Nothing is implied about the relationship between the input and output waveforms!

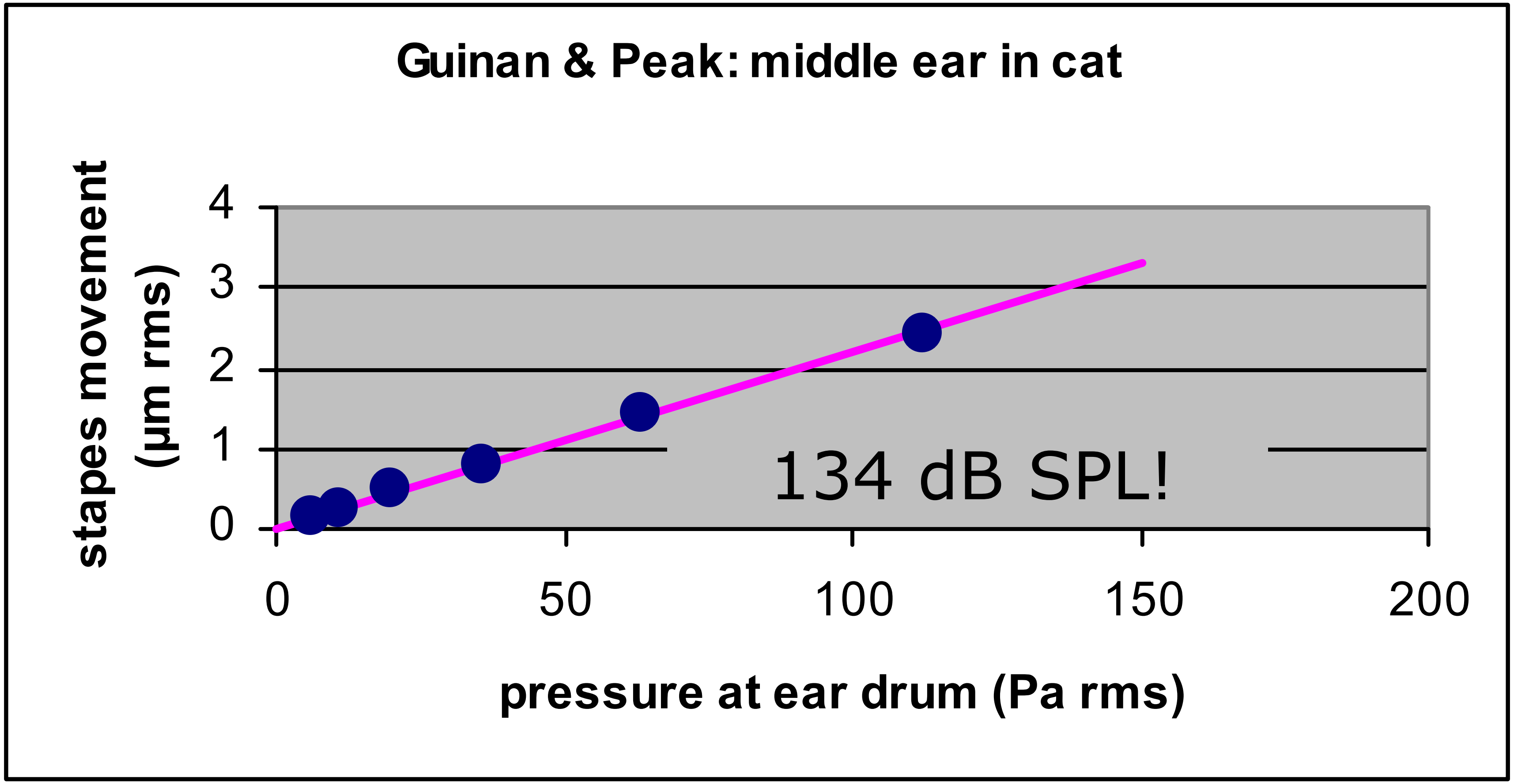
A typical test of homogeneity

- Present a sinewave of a particular frequency to a system (but it can be *any* fixed sound)
- Measure the level of the output signal as you vary the level of the input signal
- Plot the level of the output signal on the y-axis and the level of the input signal on the x-axis
  - *input/output function*
- If the input/output function is a straight line going through the origin (0,0), that behaviour is consistent with homogeneity
- Any other kind of curve means the system is *not* homogeneous and hence, cannot be linear.
- Would our perfect x2 amplifier be homogeneous?

An input/output function for a x2 amplifier

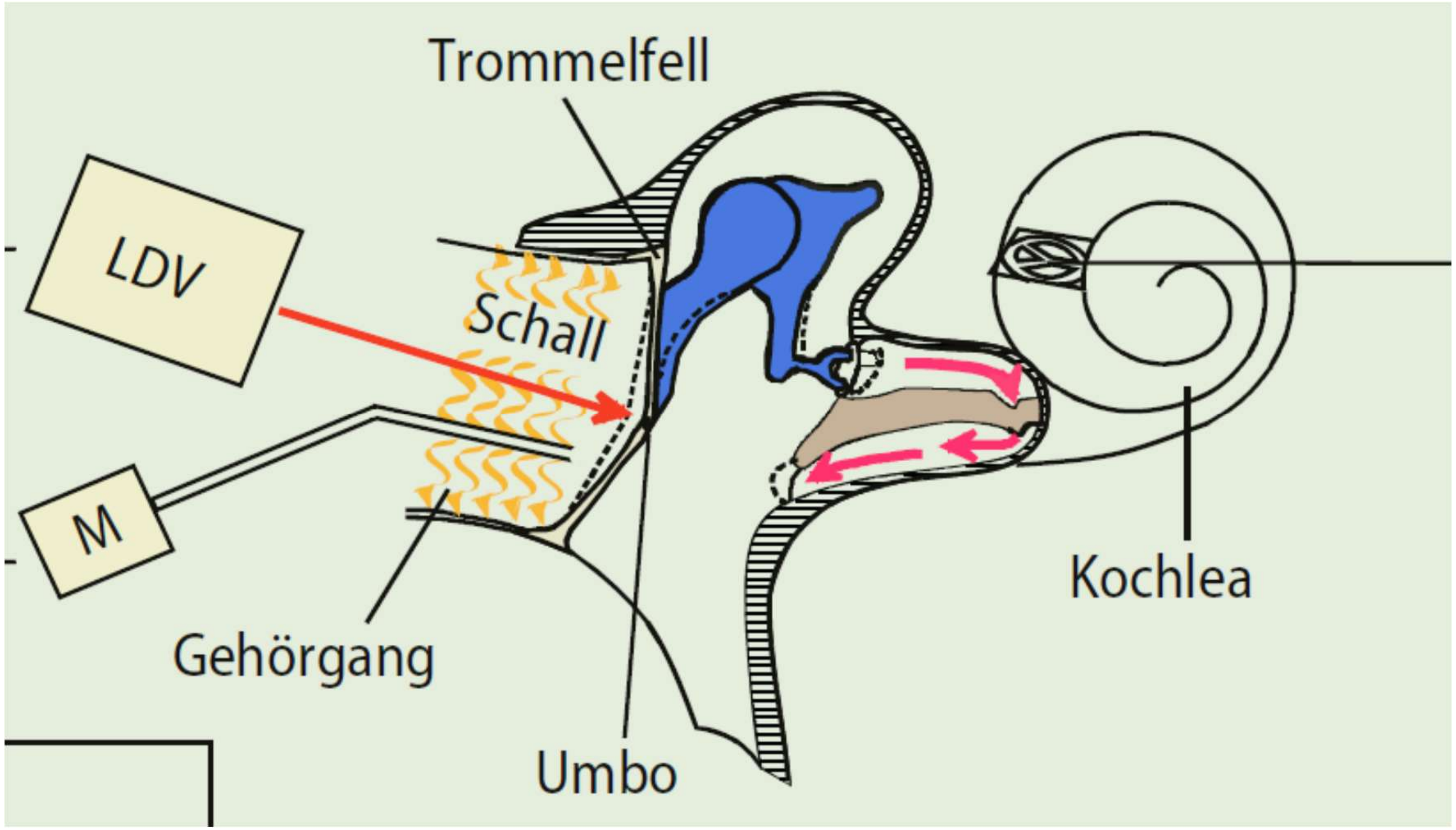


Homogeneity in the cat middle ear



Would homogeneity hold at high levels?

Laser Doppler Velocimetry of the human eardrum



what kind of
scale is this?

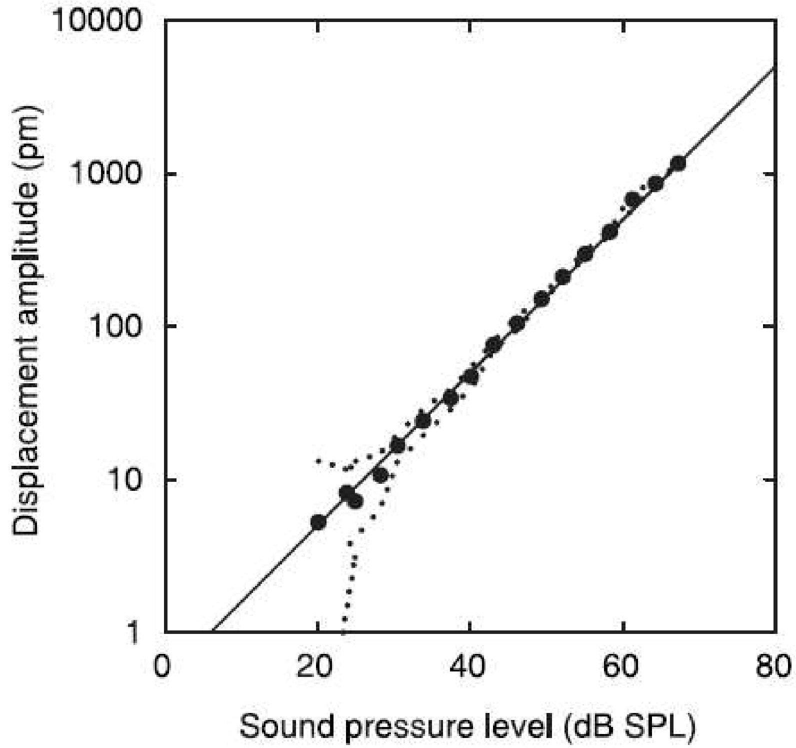
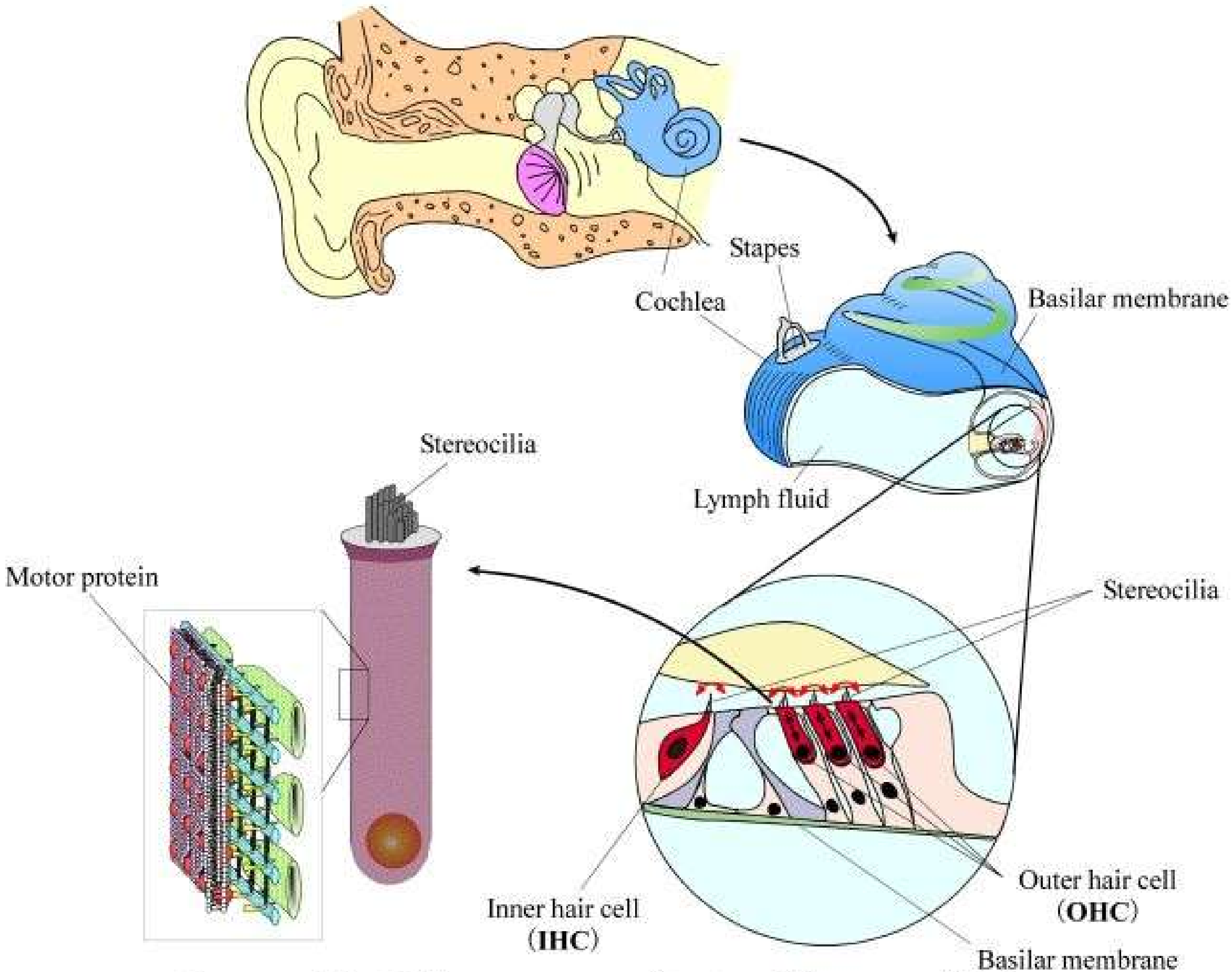


Fig. 1. Dependence of umbo displacement amplitude on SPL for single-tone stimulation (3.5 kHz) measured for an open sound field. The linear regression line of unity slope (1 dB/dB) indicates that the measured umbo response is linear. The dotted lines delineate the maximum noise level in the 100-Hz sidebands adjacent to the stimulus frequency. A reflector was not placed on the umbo. (Subject identifier: JT.)

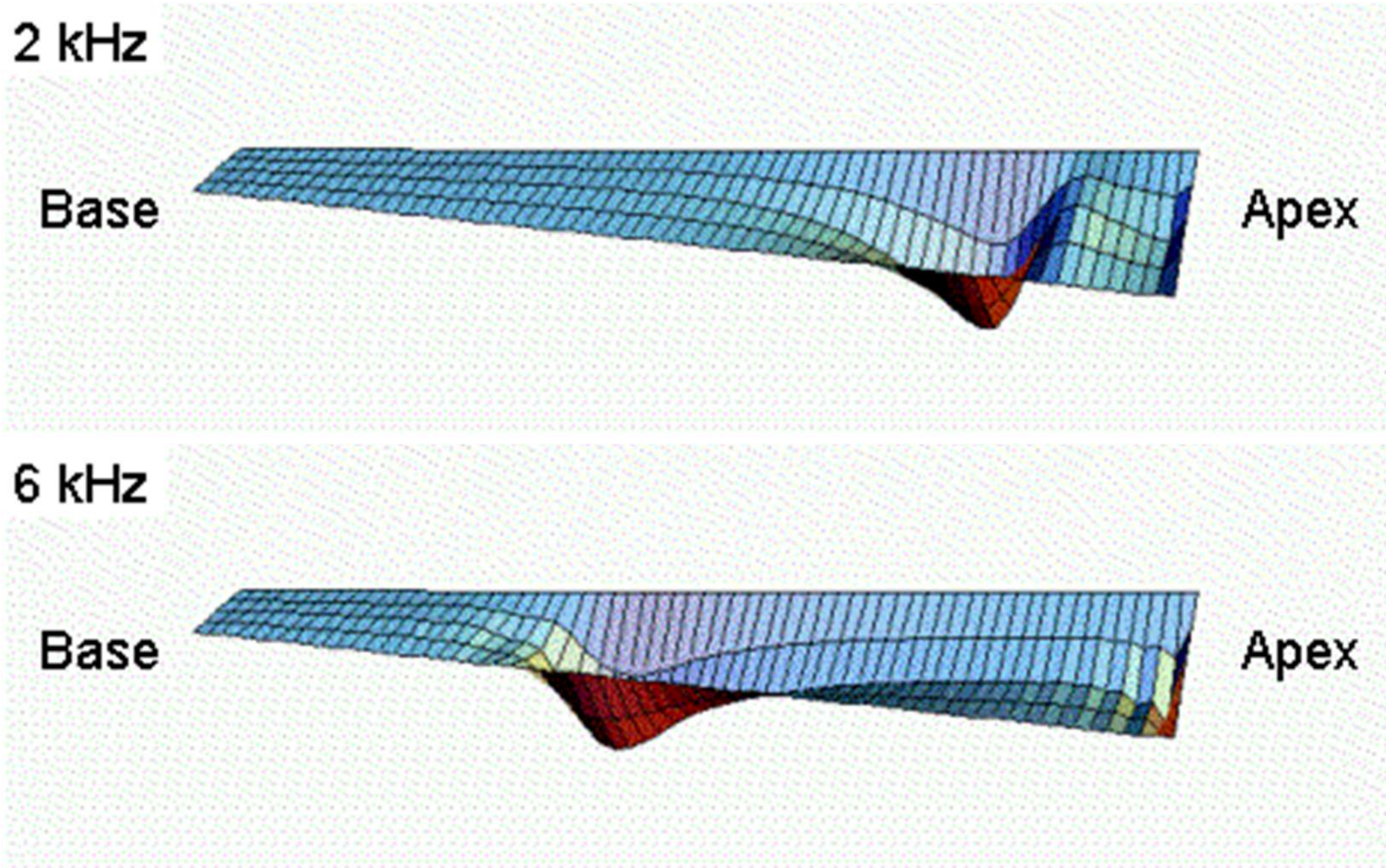
Dalhoff et al. (2007) *PNAS* 104, 1546-1551



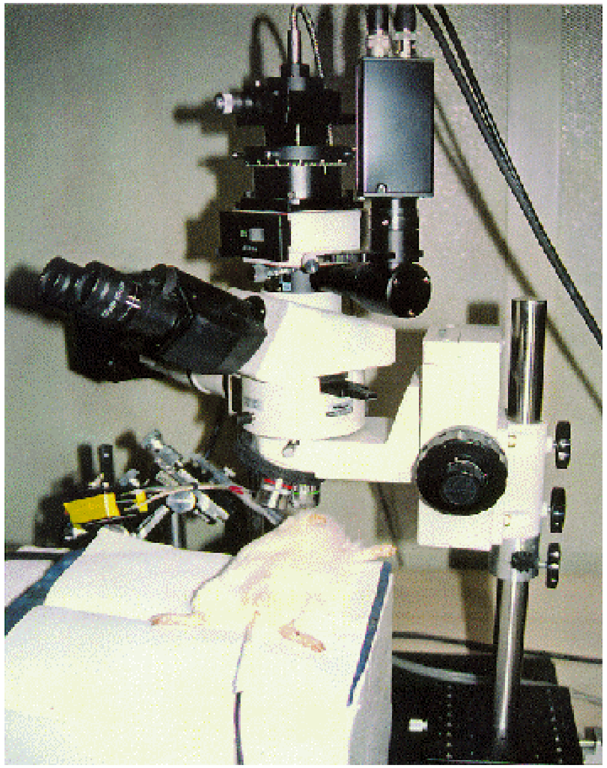
Structure of the OHC

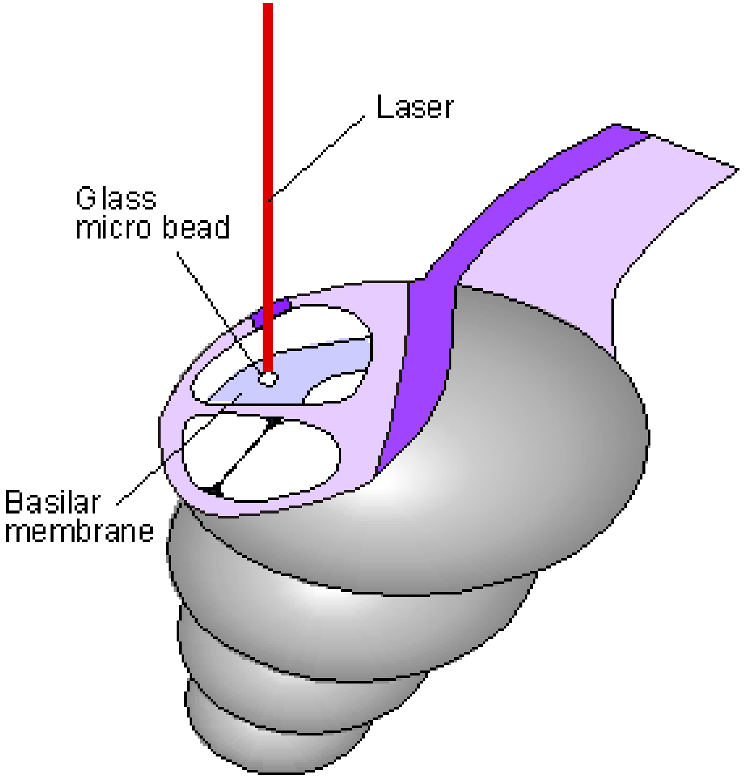
Structure of the organ of Corti

Basilar membrane motion to two sinusoids of different frequency



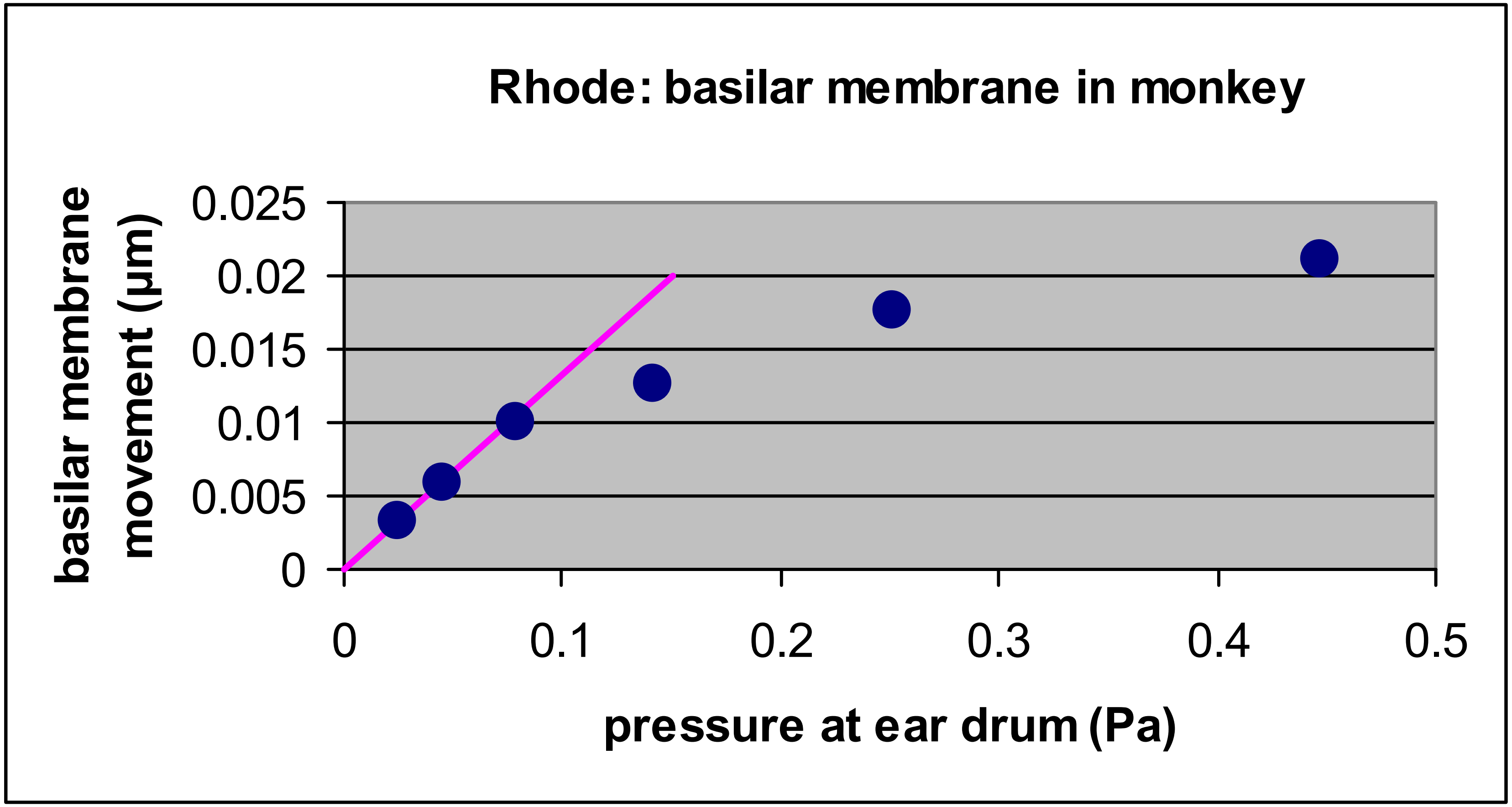
Laser Doppler Velocimetry on the basilar membrane





http://www.wadalab.mech.tohoku.ac.jp/bmldv-e.html

Homogeneity in the monkey inner ear?



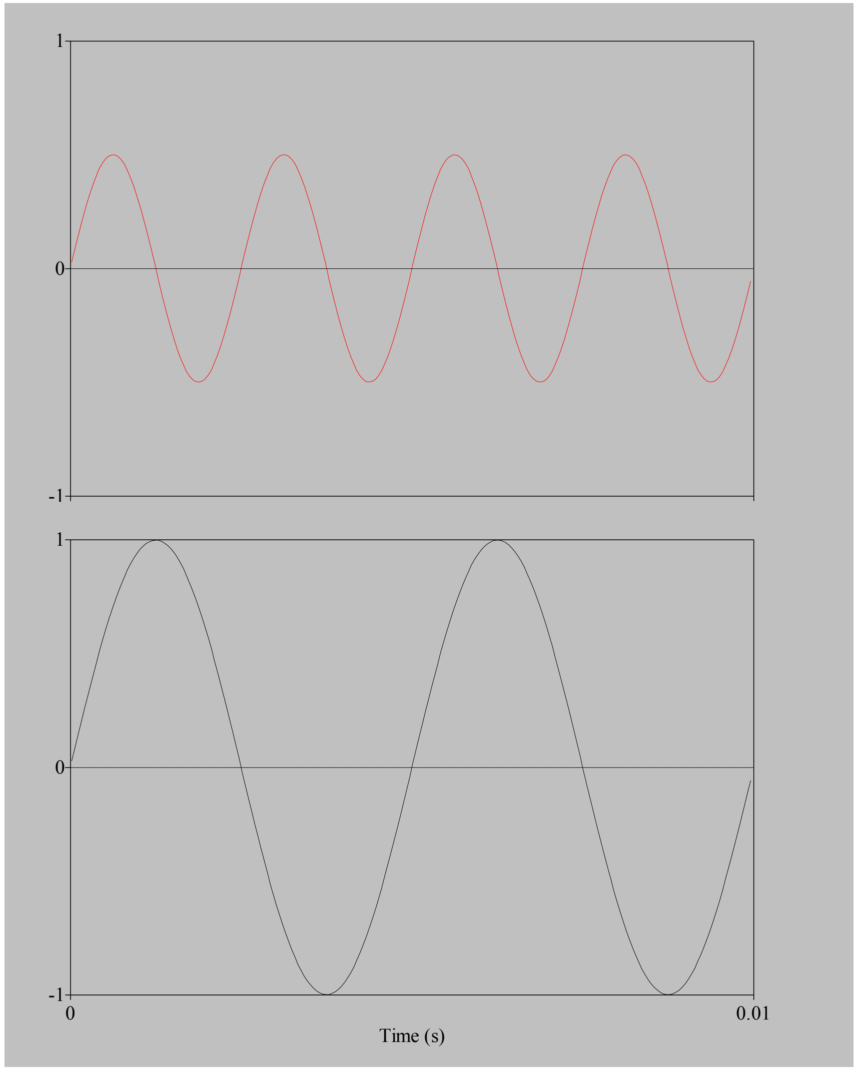
Linearity Part II: Additivity

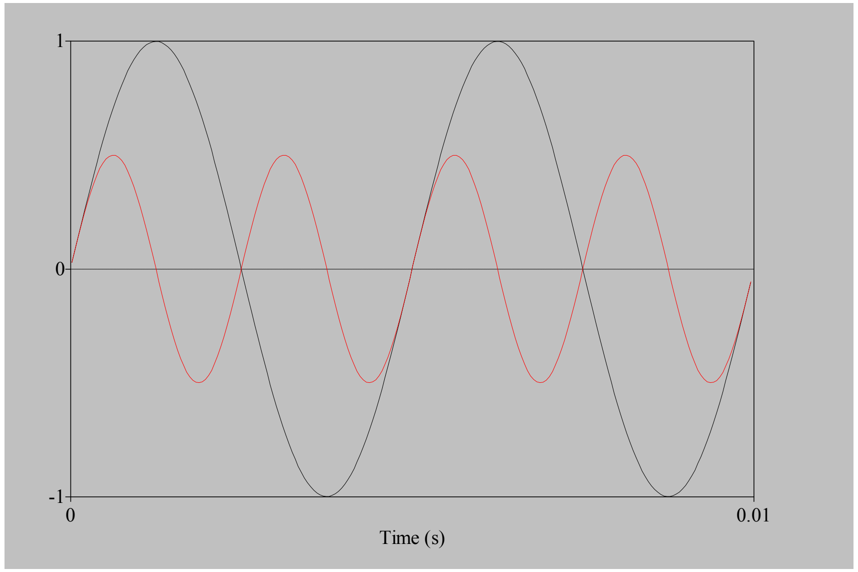
But first, what does it mean to add two waves?

Adding Waveforms

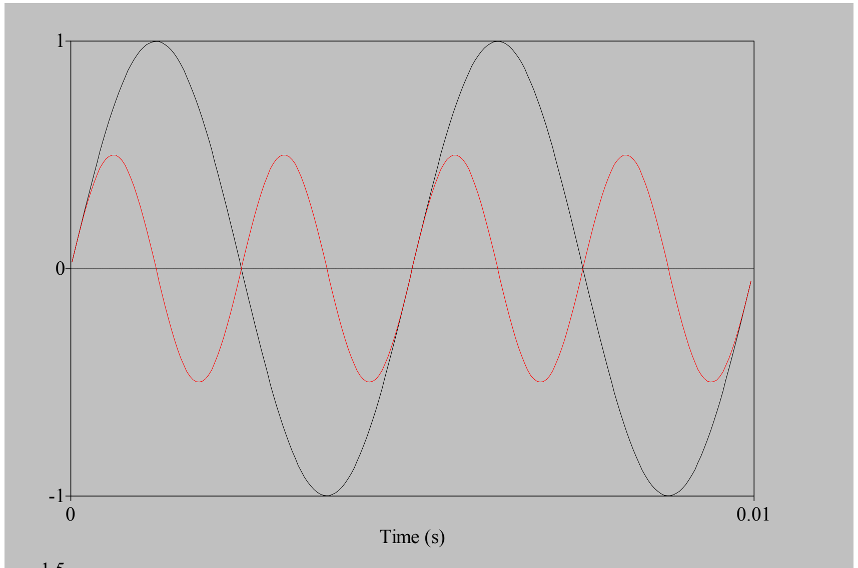
400 Hz
½ V

200 Hz
1 V



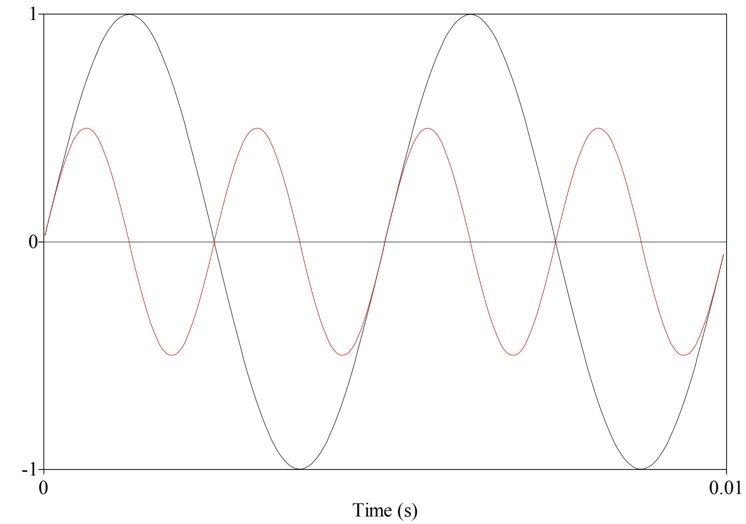

1
0
-1
0
0.01
Time (s)

Adding up the two waveforms is performed by adding the amlitude values at each point in time.



Adding Waveforms

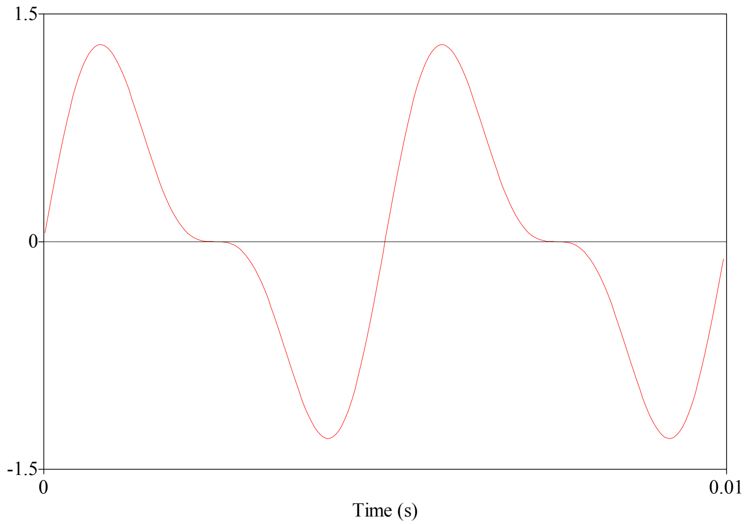
sinusoids



200 Hz

400 Hz

complex waveform

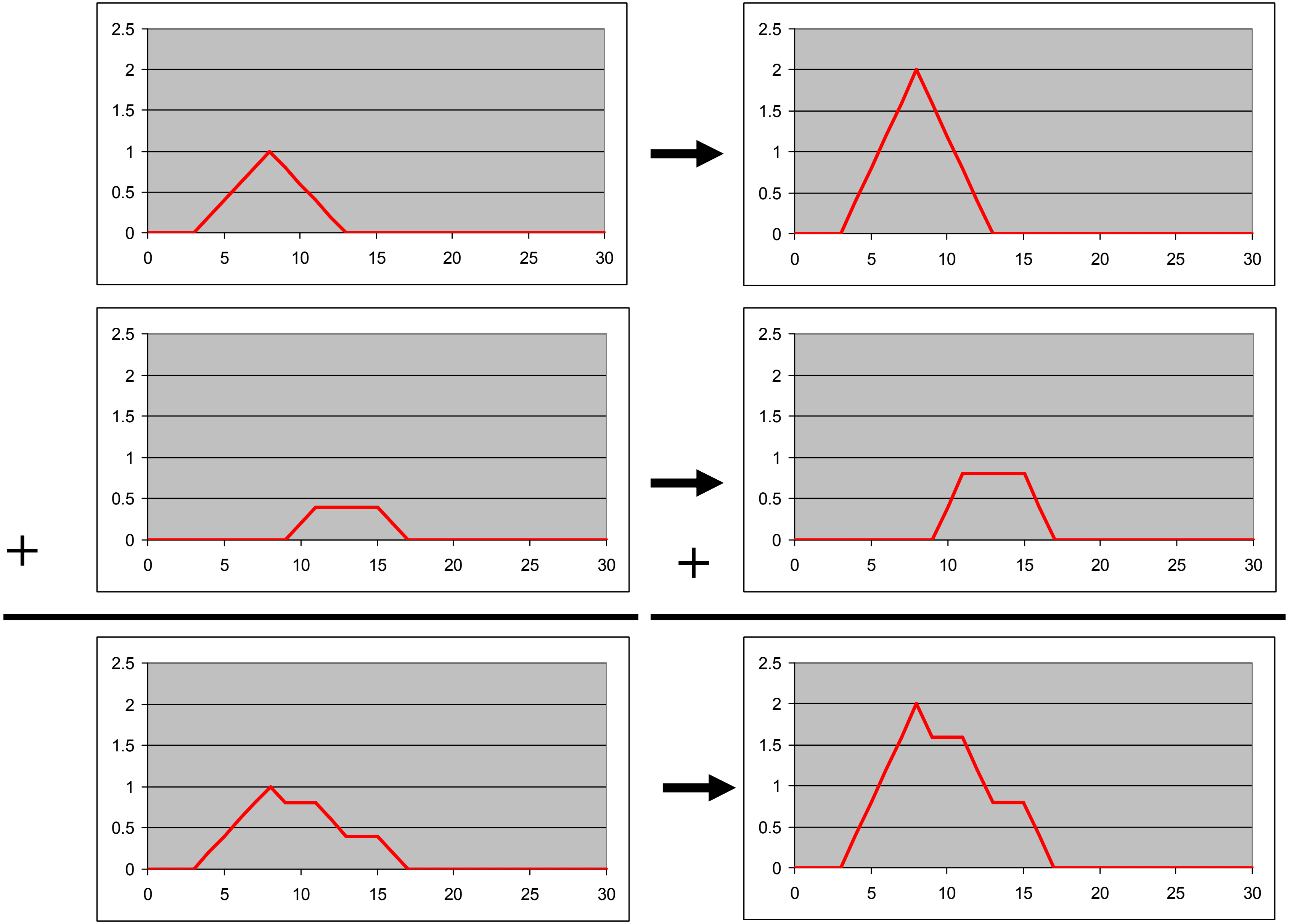


200+400 Hz

Linearity in a system: Additivity

- Additivity (principle of superposition)
- The output of a system to two input signals added together, is the same as the separate output signals for each of the inputs on their own, added together.
- In other words, signals don't interact.
- In simple equations:
  - If $inp_1(t) \rightarrow outp_1(t)$ & $inp_2(t) \rightarrow outp_2(t)$
  - Then $inp_1(t) + inp_2(t) \rightarrow outp_1(t) + outp_2(t)$

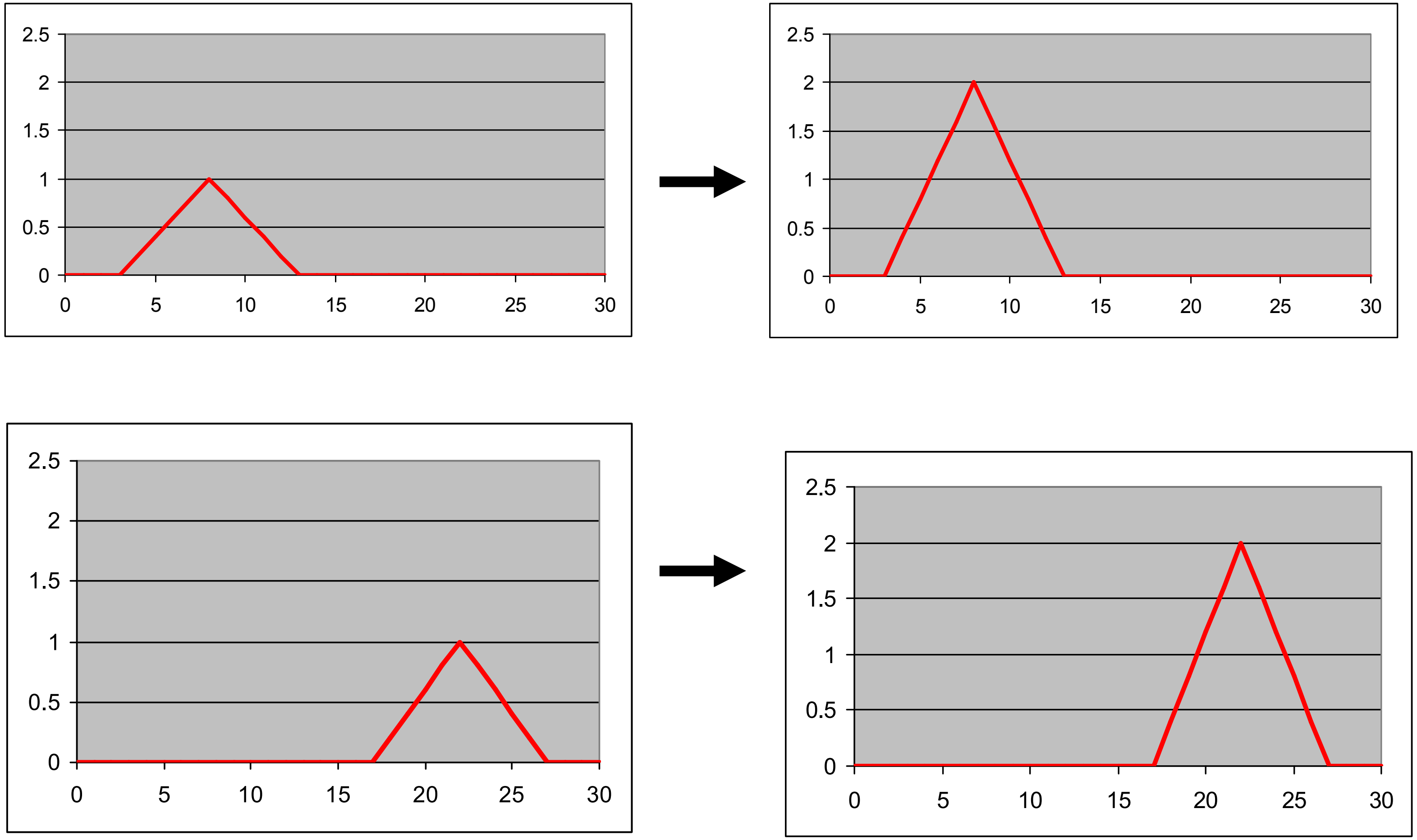
Additivity: A simple example



Requirement 3: Time-invariance

- For a particular pair of input and output signals, delaying the input signal by a particular amount also delays the output signal by the same amount.
- The system’s behavior does not change in time

Time invariance: A simple example



Our goal

To characterise the behaviour of a system that allows us to predict the output of the system to any input signal

Our motto

We don't care how a system changes a signal, we only care for what the system does to the signal, so ...

We don't study the system itself but we compare the output to the input.

LTI systems are...

... linear

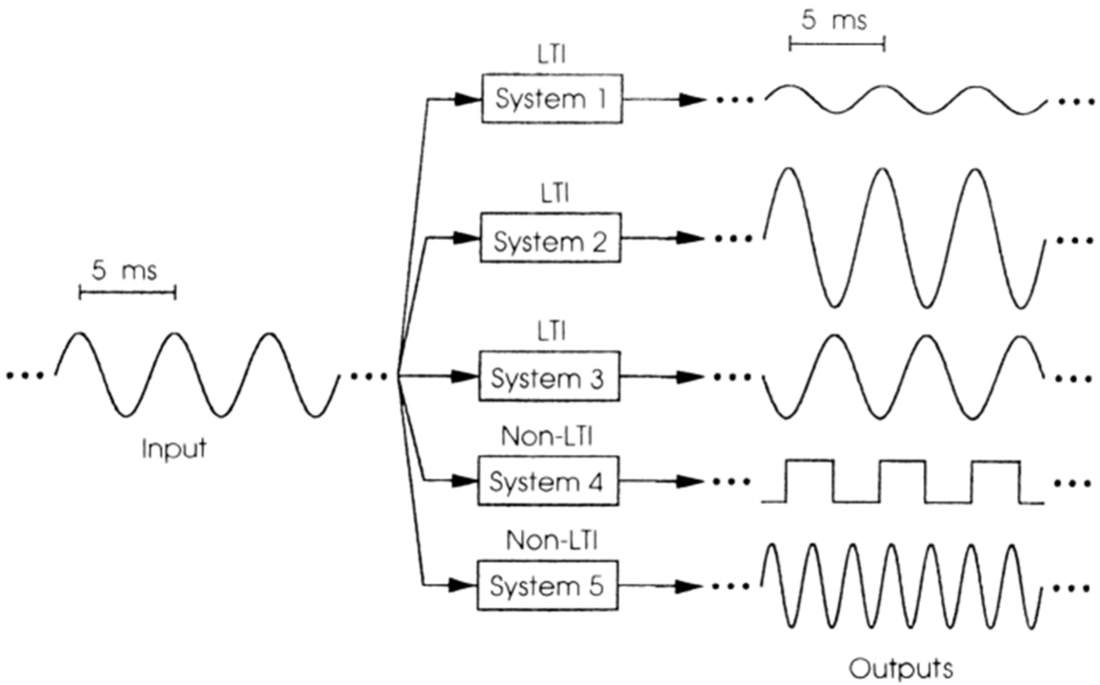
- Homogeneity
  - The amplitude of output signals grows proportionally with the amplitude of input signals, with no change in the *shape* of the output
- Additivity
  - The output to the sum of two input signals is the sum of the outputs to the two inputs separately
  - Signals don’t interact

... time-invariant

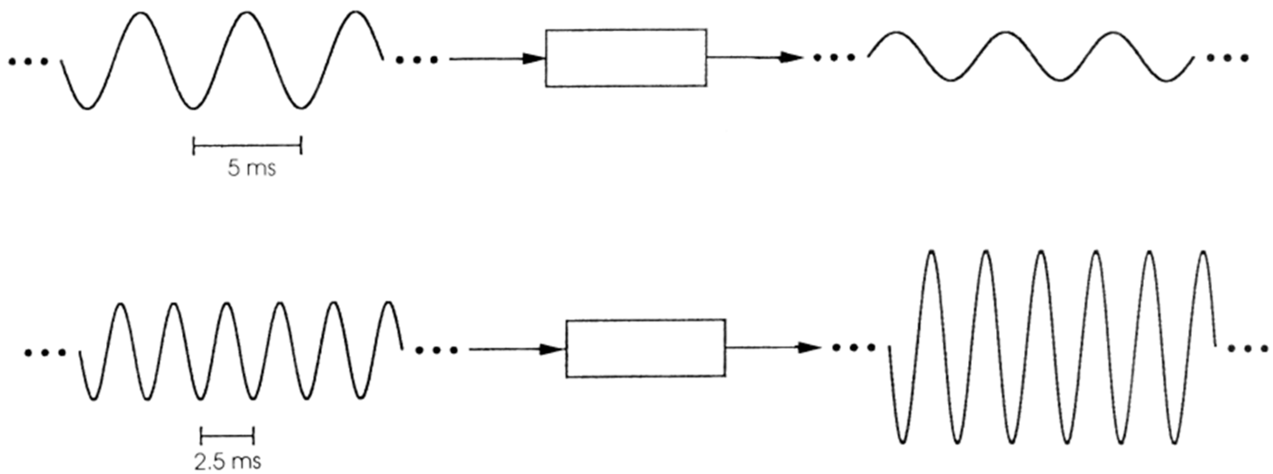
- What a system does to an input signal today, is the same as what it will do tomorrow
- The system does not change its behaviour over time

An LTI system can be completely characterised by its response to sinusoids

Sinusoidal input signals to an LTI system always lead to sinusoidal outputs of the **same frequency**

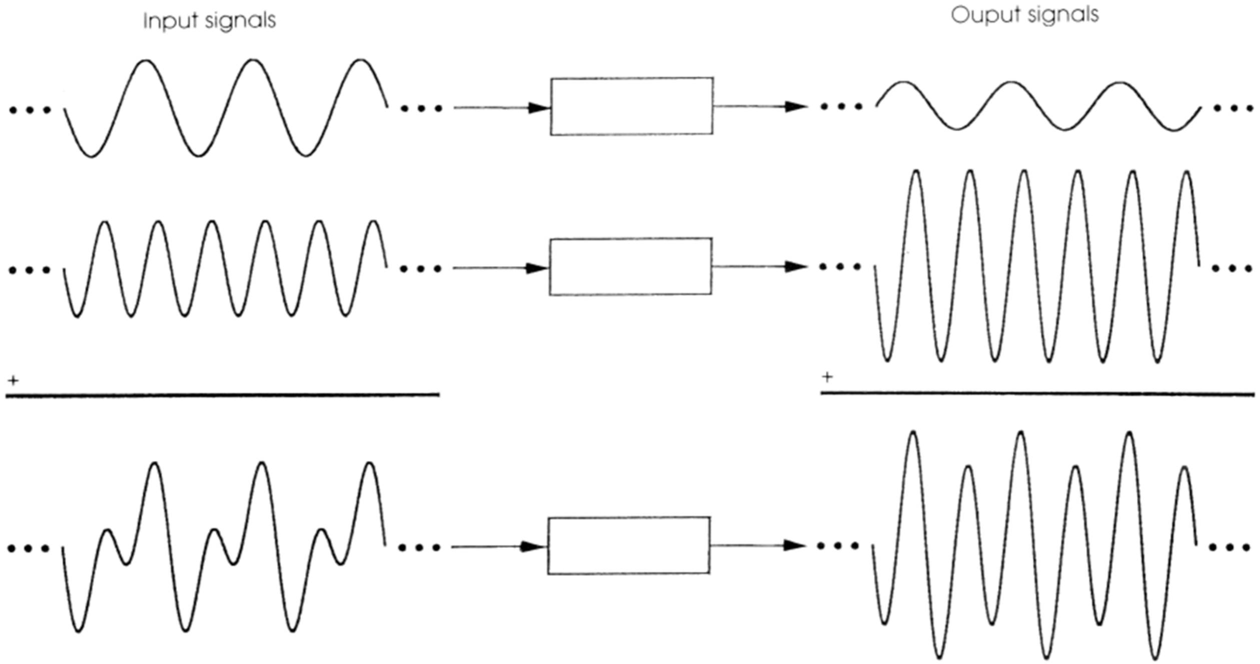


Knowing the response of a system to a sinusoid of a particular frequency, amplitude and phase allows the prediction of the output of the system to a sinusoid of the same frequency, but any amplitude and any phase



Why?

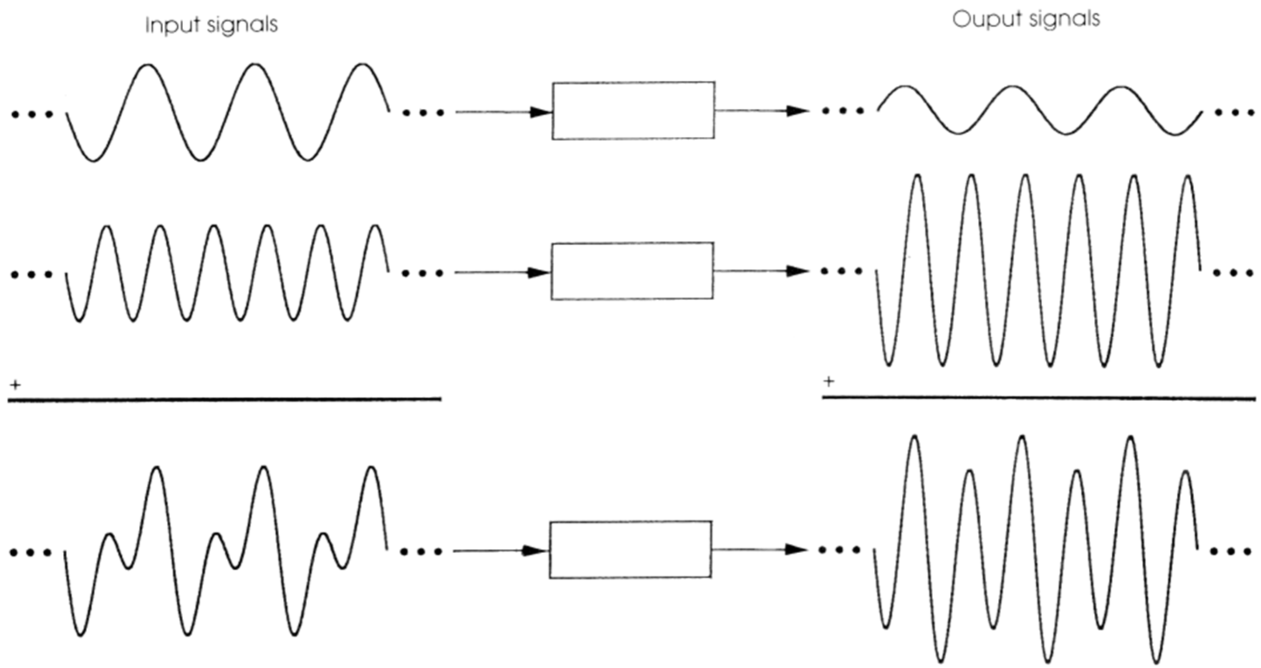
Knowing the response of a system to any frequency sinusoid allows the prediction of the output of the system to any signal that can be made from adding up sinusoids of *any* frequency, amplitude and phase



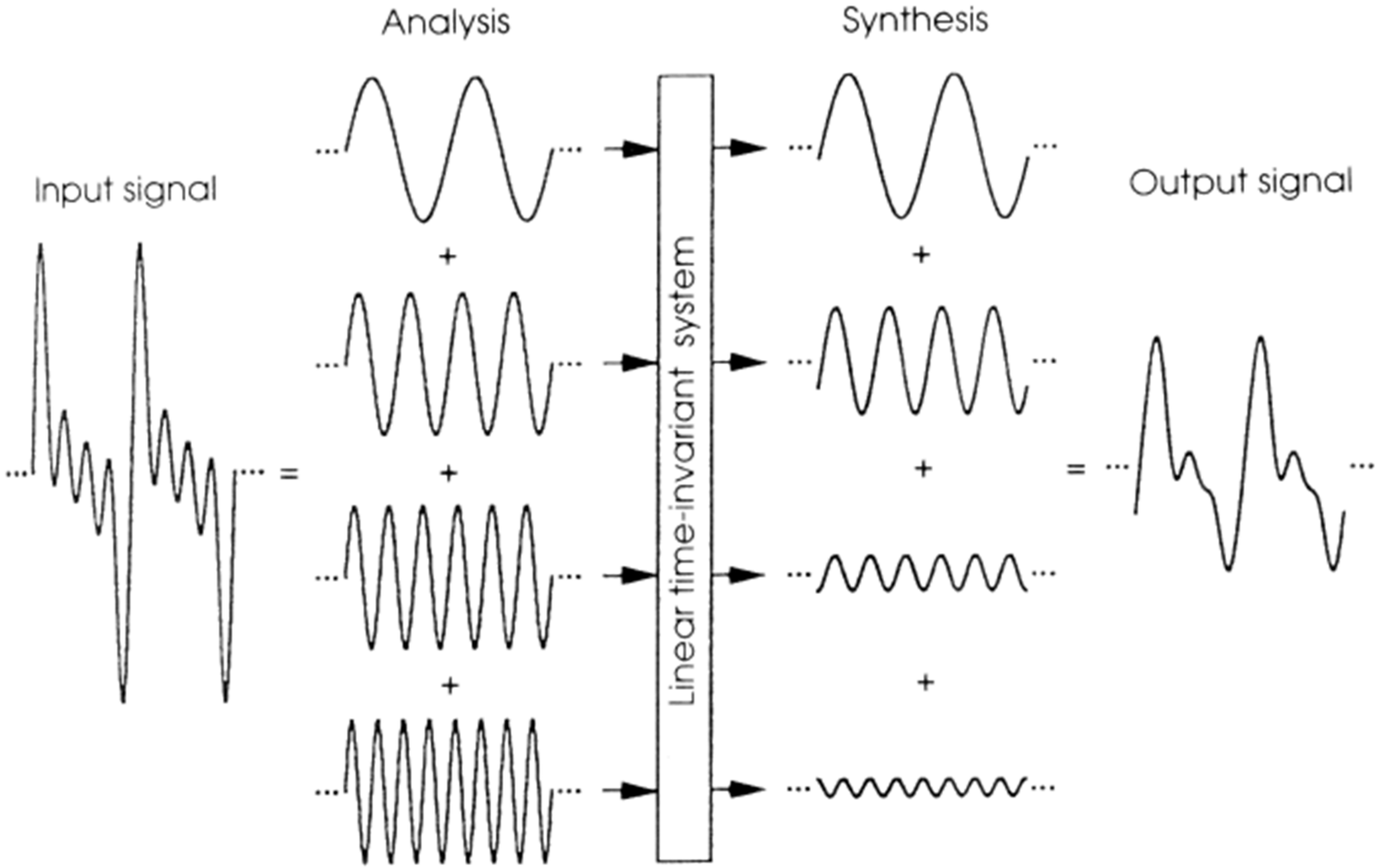
Why?

Fact number 3

Any complex wave can be made by adding up sinusoids of varying frequency, amplitude and phase



The **BIG** idea: Illustrated

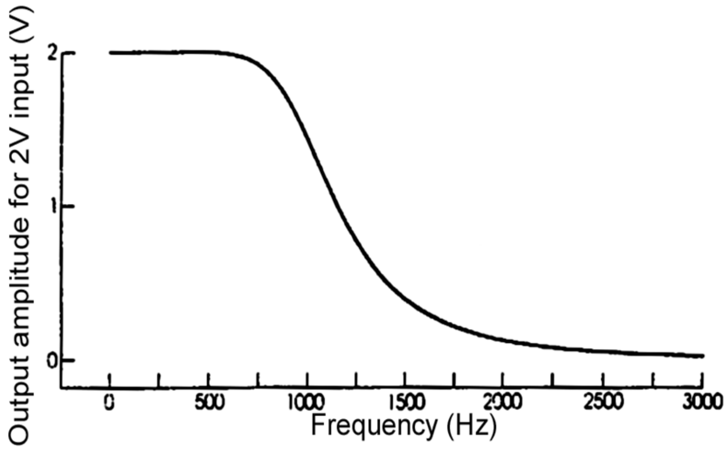


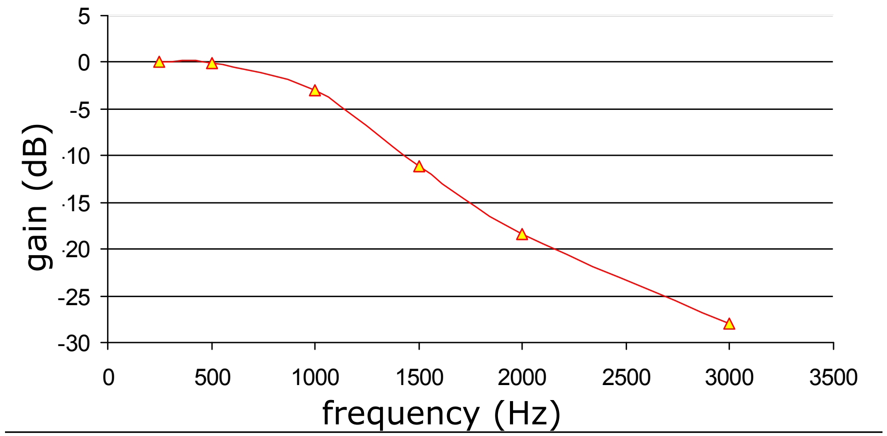
Physical systems react differently to different frequencies

- A swing or pendulum
- Acoustic resonators
- Mass on a spring
- Bridges

Tacoma Narrows Bridge Collapse.wmv

Amplitude Response: Key points





- The change made by a system to the amplitude of a sinewave, specified over a range of frequencies.
- Response = output amplitude/input amplitude
- Usually scaled in dB as:
  20 x log(output amplitude/input amplitude)
  = response (dB re input amplitude)

Once we have a frequency response

- We can calculate the output for any given sinusoidal input
- For any particular frequency, we know that
  - $response_f$ = $output\ amplitude_f$ / $input\ amplitude_f$
- So that
  - $output\ amplitude_f$ = $response_f$ x $input\ amplitude_f$
- Why is this valid?